SYNOPTIC: Calculation of Compressible Adiabatic Turbulent Boundary Layers, Tuncer Cebeci, A.M.O. Smith, and G. Mosinskis, Douglas Aircraft Company, Long Beach, Calif.; *AIAA Journal*, Vol. 8, No. 9, pp. 1974–1982.

Fluid Dynamics, Boundary Layers, and Convective Heat Transfer—Turbulent

Theme

Describes a general method for calculating turbulent boundary layers in two-dimensional and in axisymmetric flows and investigates the accuracy of the method for compressible adiabatic flows. The method is based on the ideas of eddy transport coefficients and the numerical solution of the governing equations in their differential form.

Content

The governing equations for compressible flows about two dimensional and axisymmetric bodies,

$$(\partial/\partial x)(r^k \rho u) + (\partial/\partial y)[r^k(\rho v + \langle \rho' v' \rangle)] = 0 \quad (1)$$

$$\rho u \frac{\partial u}{\partial x} + (\rho v + \langle \rho' v' \rangle) \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{1}{r^k}\frac{\partial}{\partial y}\left[r^k(\mu \frac{\partial u}{\partial y} - \rho\langle u'v' \rangle)\right] \quad (2)$$

$$\rho u \frac{\partial H}{\partial x} + (\rho v + \langle \rho' v' \rangle) \frac{\partial H}{\partial y} = \frac{1}{r^k}\frac{\partial}{\partial y}\left\{r^k\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} - \rho\langle v'H' \rangle + \mu\left(1 - \frac{1}{Pr}\right) u \frac{\partial u}{\partial y}\right]\right\} \quad (3)$$

subject to the boundary conditions

$$y = 0,\ u = 0,\ v = 0, \text{ or } v = v_w(x),$$
$$H = H_w(x), \text{ or } \partial H/\partial y = (\partial H/\partial y)_w \quad (4)$$
$$y \to \infty,\ u \to u_e(x),\ H \to H_e(x)$$

are solved numerically by an implicit finite-difference method. The time mean of the fluctuating quantities $-\rho\langle u'v' \rangle$ and $-\rho\langle v'H' \rangle$ are eliminated by means of eddy viscosity (ϵ) and eddy-conductivity (λ_t) concepts, respectively.

$$-\rho\langle u'v' \rangle = \rho\epsilon(\partial u/\partial y) \quad (5)$$

$$-\rho\langle v'H' \rangle = \rho\lambda_t(\partial H/\partial y) \quad (6)$$

A large number of flows computed by this method show that the method is quite accurate, as it was in incompressible flows with and without heat and mass transfer.[1–3] The velocity profiles, Mach profiles as well as local skin-friction coefficients, agree quite well with experiments for a range of Mach number up to 5; the rms error in calculated local skin friction values is 3.5%, based on 43 experimental values obtained by the floating element technique. Figures 1 and 2 show comparisons of calculated and experimental results for two of the many flows computed by this method.

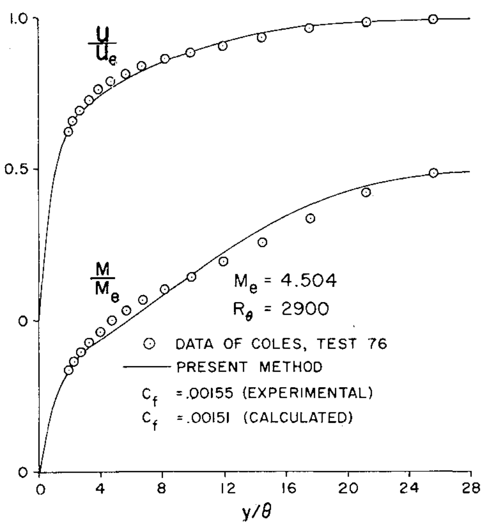

Fig. 1 Comparison of results for the flat-plate flow measured by Coles.[4] Skin friction was measured by floating element.

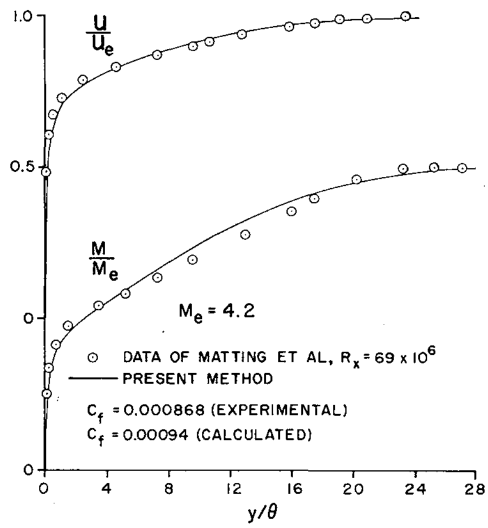

Fig. 2 Comparison of results for the flatplate flow measured by Matting et al.[5] Skin friction was measured by floating element.

References

[1] Cebeci, T. and Smith, A. M. O., "A Finite Difference Solution of the Incompressible Turbulent Boundary Layer Equations by an Eddy-Viscosity Concept." *Proceedings of the 1968 AFOSR-IFP-Stanford Conference on Computation of Turbulent Boundary Layers*, Vol. 1, Stanford University, pp. 346–355.

[2] Cebeci, T., Smith, A. M. O., and Mosinskis, G., "Solution of the Incompressible Turbulent Boundary Layer Equations with Heat Transfer." *Journal of Heat Transfer*, Vol. 92, No. 1, Feb. 1970, pp. 133–143.

[3] Cebeci, T. and Mosinskis, G., "Prediction of Turbulent Boundary Layers with Mass Addition, Including Highly Accelerating Flows," to be presented to the ASME 1970 Space Technology and Heat Transfer Conference, Los Angeles, Calif., June 1970.

[4] Coles, D. E., "Measurements in the Boundary Layer on a Smooth Flat Plate in Supersonic Flow, III. Measurements in a Flat Plate Boundary Layer at the Jet Propulsion Laboratory." Rept. 20-71, June 1953, Jet Propulsion Lab., California Institute of Technology, Pasadena, Calif.

[5] Matting, F. W. et al., "Turbulent Skin Friction at High Mach Numbers and Reynolds Numbers in Air and Helium," TR R-82, 1961, NASA.